OPTIMAL Y-U-V MODEL BASED ON KARHUNEN-LOEVE TRANSFORMATION

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ABSTRACT

An optimal Y-U-V transformation based on Karhunen-Loeve transformation for image compression proposed in this paper is considered as a spectral redundancy reduction. The PSNR is gained for optimal Y-U-V in comparison with traditional fixed Y-U-V transformation, because the variances are most separately after K-L transformation and the down sampling is taken on the coordinates with smallest variances in optimal Y-U-V transformation. The K-L transformation in optimal Y-U-V is an image-dependent transform that is to de-correlate the data in color spectral domain by using eigenvector matrix of covariance of the colors of the image. A normalization matrix follows the optimal Y-U-V transformation is used for ranging the data of Y-U-V within 0-255. The procedure is used in encoding only and the Y-U-V transformation can be transmitted with compressed data and de-coding easily.

Keywords: *Y-U-V transformation, statistical redundancy, down sampling, Karhunen-Loeve transformation*

1 INTRODUCTION

In color image processing, each pixel represents in intensities of red, green, and blue components separately. It is known that the most color images are recorded in R-G-B domain and are transformed to Y-U-V or Y-Cr-Cb for transmission of color image and video signals [1]. Because the data of two transformation matrices are very similar, the Y-U-V color coordinate system is adopted only, which is named as "Fixed Y-U-V" in this paper.

$$\begin{bmatrix} Y \\ U \\ V \end{bmatrix} = T_{YUV} \begin{bmatrix} R \\ G \\ B \end{bmatrix}, \quad T_{YUV} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.148 & -0.289 & 0.437 \\ 0.615 & -0.515 & -0.100 \end{bmatrix} \qquad (1)$$

$$\begin{bmatrix} Y \\ Cr \\ Cb \end{bmatrix} = T_{YCrCb} \begin{bmatrix} R \\ G \\ B \end{bmatrix}, \quad T_{YCrCb} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.169 & -0.331 & 0.500 \\ 0.500 & -0.419 & -0.081 \end{bmatrix} \qquad (2)$$

The Y-U-V transformation includes both transformation operation and down sampling operation.

2 Y-U-V MODEL ON KARHUNEN-LOEVE TRANSFORMATION

The color image transform from R-G-B coordinates into Y-U-V coordinates can be considered as color multi-spectral data transform [2]. The statistical redundancy includes both spectral and spatial redundancy. The Y-U-V transformation is considered as a spectral redundancy reduction. The spatial redundancy reduction can be used such as DCT or wavelets after Y-U-V transformation. But we only focus on the Y-U-V transformation in this paper. The traditional fixed Y-U-V transformation does not depend on the image, so the information of the image is not fully utilized. An optimal Y-U-V transformation based on Karhunen-Loeve transformation is proposed in this paper.

In Karhunen-Loeve transformation, the effectiveness of each feature is determined by its corresponding eigenvalue, which is equal to variance. In feature selection, the mean square error is minimal if the features with the smallest eigenvalue are deleted. The optimal Y-U-V transformation based on Karhunen-Loeve transformation is that the mean square error is minimal if the features with the smaller eigenvalue are down sampled. If more down sample is taken for the smaller eigenvalue of feature, the result will be better.

The benefit is gained for optimal Y-U-V in comparison with traditional fixed Y-U-V transformation, because the variances are most separately and the down sampling is taken on the coordinates with smallest variances. The K-L transformation in optimal Y-U-V is an image-dependent transform that is to de-correlate the data in color spectral domain by using eigenvector matrix of covariance of the colors of the image. The optimal Y-U-V and the fixed Y-U-V are shown Fig. 1 and Fig. 2 respectively.

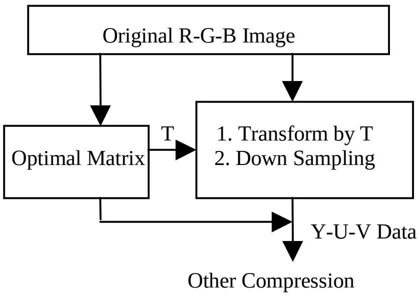


Fig. 1 Optimal Transformation of Y-U-V

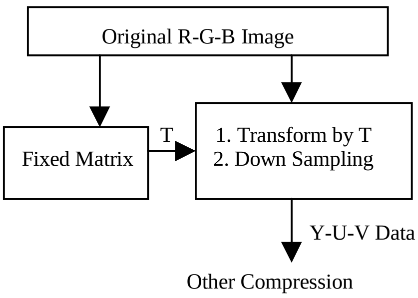


Fig. 2 Traditional Fixed Transformation of Y-U-V

The transformation of R-G-B to Y-U-V can also be considered as an orthogonal transformation and followed a normalization procedure.

$$\begin{bmatrix} Y \\ U \\ V \end{bmatrix} = \begin{bmatrix} s_Y(y+t_Y) \\ s_U(u+t_U) \\ s_V(v+t_V) \end{bmatrix}, \quad \begin{bmatrix} y \\ u \\ v \end{bmatrix} = T \begin{bmatrix} R \\ G \\ B \end{bmatrix} \qquad (3)$$

The normalization of both translation and scaling operations are getting each 1D data within 0-255. The normalization matrix :

$$\eta = \begin{bmatrix} t_Y & s_Y \\ t_U & s_U \\ t_V & s_V \end{bmatrix} = \begin{bmatrix} \text{Translation} & \text{Scaling} \end{bmatrix} \qquad (4)$$

The transformation matrix T is an eigenvector matrix of 3 by 3 covariance matrix C of R-G-B color image, where $E\{\cdot\}$ indicates ensemble average and the superscript " $'$ " denotes transpose operation. The transformation from R-G-B to Optimal Y-U-V can be considered as a diagonalization transform of the covariance matrix C. The elements of diagonal matrix Λ are the variance of Y, U and V respectively.

Covariance Matrix:

$$C = E\{(X-M)(X-M)'\} \qquad (5)$$

Orthogonal Transformation by Eigenvector Matrix T:

$$TCT' = \Lambda \qquad (6)$$

Where Orthogonal Transformation

$$T'T = I \qquad (7)$$

Eigenvalue Matrix

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \qquad (8)$$

Random Vector and its Mean

$$X = \begin{bmatrix} R \\ G \\ B \end{bmatrix}, \quad M = E\left\{\begin{bmatrix} R \\ G \\ B \end{bmatrix}\right\} \qquad (9)$$

The PSNR (Peak Signal-to-Noise Ratio) is used for describing the color image compression:

$$\text{PSNR} = 10 \cdot \log_{10}\left(\frac{255^2}{\frac{1}{3 \cdot N_X \cdot N_Y} \sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} \sum_{k=1}^{3} \left(f_{rec,i,j,k} - f_{ori,i,j,k} \right)^2} \right) \qquad (10)$$

Where:

N_X, N_Y Is the size of image, for example 512×512 for Lena image

$f_{ori,i,j,k}$ is pixel gray level of original image (k=1 for red, k=2 for green and k=3 for blue image)

$f_{rec,i,j,k}$ is pixel gray level of recovered image after YUV transformation and de-transformation.

The computation results of transformation and normalization matrices of optimal Y-U-V of images are listed:

3 EXAMPLES

3.1 Lena 512×512

Orthogonal transformation matrix:

$$T_{\text{lena}} = \begin{bmatrix} 0.6775 & 0.6193 & 0.3968 \\ -0.6723 & 0.3025 & 0.6757 \\ 0.2984 & -0.7245 & 0.6213 \end{bmatrix} \tag{11}$$

Normalization matrices:

$$\eta_{\text{lena}}^{(4-1-1)} = \begin{bmatrix} 0 & 0.6007 \\ 62 & 1 \\ 6 & 1 \end{bmatrix}, \eta_{\text{lena}}^{(16-1-1)} = \begin{bmatrix} 0 & 0.6007 \\ 56 & 1 \\ 6 & 1 \end{bmatrix}$$
$$\eta_{\text{lena}}^{(256-16-1)} = \begin{bmatrix} 0 & 0.6007 \\ 56 & 1 \\ 6 & 1 \end{bmatrix} \tag{12}$$

3.2 Lena 64×64

Orthogonal transformation matrix:

$$T_{\text{le64}} = \begin{bmatrix} 0.5080 & 0.6600 & 0.5534 \\ 0.8410 & -0.2414 & -0.4841 \\ 0.1860 & -0.7114 & 0.6678 \end{bmatrix} \tag{13}$$

Normalization matrices:

$$\eta_{\text{le64}}^{(4-1-1)} = \begin{bmatrix} 0 & 0.6173 \\ 0 & 1 \\ 6 & 1 \end{bmatrix}, \eta_{\text{le64}}^{(16-1-1)} = \begin{bmatrix} 0 & 0.6173 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$
$$\eta_{\text{le64}}^{(256-16-1)} = \begin{bmatrix} 0 & 0.6173 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \tag{14}$$

3.3 Cow 512×512

Orthogonal transformation matrix:

$$T_{\text{cow}} = \begin{bmatrix} 0.5626 & 0.5159 & 0.6431 \\ -0.4079 & -0.5022 & 0.7625 \\ 0.7191 & -0.6913 & -0.0707 \end{bmatrix} \tag{15}$$

Normalization matrices:

$$\eta_{\text{cow}}^{(4-1-1)} = \begin{bmatrix} 0 & 0.6091 \\ 139 & 1 \\ 35 & 1 \end{bmatrix}, \eta_{\text{cow}}^{(16-1-1)} = \begin{bmatrix} 0 & 0.6091 \\ 127 & 1 \\ 35 & 1 \end{bmatrix}$$
$$\eta_{\text{cow}}^{(256-16-1)} = \begin{bmatrix} 0 & 0.6091 \\ 127 & 1 \\ 31 & 1 \end{bmatrix} \tag{16}$$

3.4 Wolf 512×768

Orthogonal transformation matrix:

$$T_{\text{wolf}} = \begin{bmatrix} 0.5435 & 0.5186 & 0.6601 \\ -0.6572 & -0.2263 & 0.7189 \\ 0.5222 & -0.8245 & 0.2179 \end{bmatrix} \tag{17}$$

Normalization matrices:

$$\eta_{\text{wolf}}^{(4-1-1)} = \begin{bmatrix} 0 & 0.6105 \\ 89 & 1 \\ 21 & 1 \end{bmatrix}, \eta_{\text{wolf}}^{(16-1-1)} = \begin{bmatrix} 0 & 0.6105 \\ 86 & 1 \\ 20 & 1 \end{bmatrix}$$
$$\eta_{\text{wolf}}^{(256-16-1)} = \begin{bmatrix} 0 & 0.6105 \\ 86 & 1 \\ 18 & 1 \end{bmatrix} \tag{18}$$

3.5 Table

Table 1 Results of Optimal Y-U-V and Fixed Y-U-V

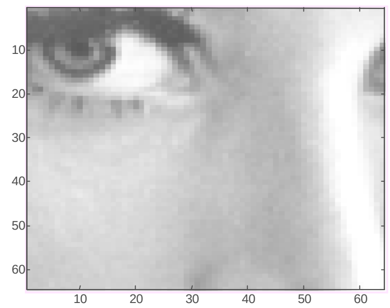
Down Sample	Variance ($\lambda_1, \lambda_2, \lambda_3$)			PSNR	D
	Y	U	V	dB	dB
(a) Lena 512×512					
4-1-1 (optimal)	2745	787	59	35	0
4-1-1 (fixed)	2850	169	366	35	-
16-1-1 (optimal)	2745	757	53	33	0
16-1-1 (fixed)	2850	164	356	33	-
256-16-1 (optimal)	2745	757	42	33	2
256-16-1 (fixed)	2850	164	319	31	-
(b) Le64 64×64					
4-1-1 (optimal)	1931	538	17	37	1
4-1-1 (fixed)	1844	43	296	36	-
16-1-1 (optimal)	1931	363	13	34	1
16-1-1 (fixed)	1844	39	169	33	-
256-16-1 (optimal)	1931	509	5	32	3
256-16-1 (fixed)	1844	30	280	29	-
(c) Cow 512×512					
4-1-1 (optimal)	3909	382	59	46	2
4-1-1 (fixed)	3192	135	42	44	-
16-1-1 (optimal)	3909	374	58	42	2
16-1-1 (fixed)	3192	133	41	40	-
256-16-1 (optimal)	3909	374	52	40	2
256-16-1 (fixed)	3192	133	37	38	-
(d) Wolf 512×768					
4-1-1 (optimal)	3716	160	4	47	2
4-1-1 (fixed)	2941	70	23	45	-
16-1-1 (optimal)	3716	160	4	44	1
16-1-1 (fixed)	2941	70	23	43	-
256-16-1 (optimal)	3716	160	4	43	2
256-16-1 (fixed)	2941	70	23	41	-

3.6 Comments

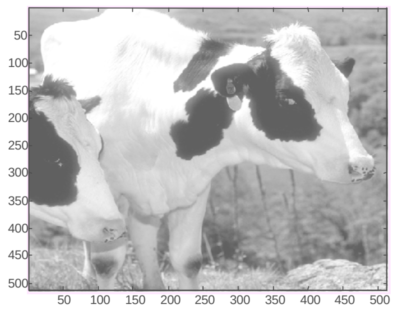
In Table 1, D represents PSNR difference between optimal Y-U-V and fixed Y-U-V transformation. To simplify the comparison, we assume that the lossless is adopted at other compression and decompression.

The "4-1-1" means that Y is not down sampled, and U and V are down sampled with 4 (with 2 both in horizontal and vertical axis). The "16-1-1" means that Y is not down sampled, and U and V are down sampled with 16 (with 4 both in horizontal and vertical axis). The "256-16-1" means that Y is not down sampled, and U is down sampled with 16 (with 4 both in horizontal and vertical axis), and V is down sampled with 256 (with 16 both in horizontal and vertical axis).

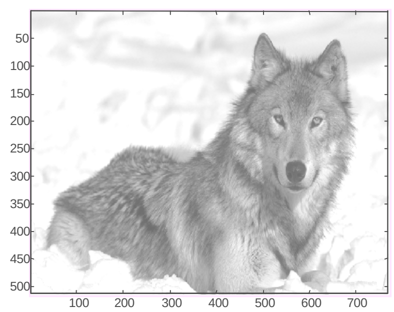
(a) Lena



(b) Lena64



(c) Cow



d) Wolf

Fig. 3 Images for Y-U-V transformation

4 SUMMARY AND CONCLUSIONS

4.1 The Y-U-V transformation proposed is optimal, because the variances are most separately after K-L transformation and the down sampling is taken on the coordinates with smallest variances in optimal Y-U-V transformation.

4.2 The optimal Y-U-V transformation obtains PSNR gain by comparing with fixed Y-U-V transformation for all images except "Lena" in down sampling 4-1-1 or 16-1-1,where D=0 (see Table 1). The fixed Y-U-V transformation matrix is probably only fine-tuned by using "Lena" in down sampling 4-1-1 or 16-1-1.

4.3 If the down sampling is 256-16-1 instead of 4-1-1 or 16-1-1,more PSNR gain can be obtained. The optimal Y-U-V transformation obtains PSNR gain by comparing with fixed Y-U-V transformation for all images in down sampling 256-16-1.

4.4 The computation of eigenvector does not take too much time and can be used for real time processing, because the K-L transformation is calculated for a 3 by 3 covariance matrix in encoding and about a dozen data need to transmit for de-coding.

4.5 The Y image includes more information in optimal Y-U-V, which can be used for printing color image by a non-color printer.

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5 REFERENCES

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